

Generalised Inaccuracy Measure and its Application in Multi-Criteria Decision Making Under Intuitionistic Fuzzy Environment.

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ABSTRACT—As a generation of ordinary fuzzy set, the concept of intuitionistic fuzzy set (IFS), characterized both by a membership degree and by a non-membership degree, is a more flexible way to cope with the uncertainty. Similarity measures of intuitionistic fuzzy sets are used to indicate the similarity degree between intuitionistic fuzzy sets. Although many similarity measures for intuitionistic fuzzy sets have been proposed in previous studies, some of those cannot satisfy the axioms of similarity or provide counterintuitive cases. In this paper, a new similarity measure and weighted similarity measure between IFSs are proposed. It proves that the proposed similarity measures satisfy the properties of the axiomatic definition for similarity measures. Comparison between the previous similarity measures and the proposed similarity measure indicates that the proposed similarity measure does not provide any counter-intuitive cases. Moreover, it is demonstrated that the proposed similarity measure is capable of discriminating difference between patterns.

Keywords: Intuitionistic fuzzy set (IFS), Similarity measures

1. INTRODUCTION

Multiple criteria decision making is concerned with creating a structure and solving the decision making problems that involve multiple criteria. The purpose is to make the lives of decision makers facing such problems a tad bit easier. We do not have a unique, optimal solution for such problems and we have to consider the decision-maker's preferences in order to differentiate between solutions.

A well thought out structure and analysis of multiple criteria explicitly leads to more informed and better decisions. A variety of approaches and methods (that have been implemented by specialized decision making software) have been developed as they have a vast array of applications ranging from medical to politics to traffic control.

This uncertainty in the decision making process can be greatly solved with the help of fuzzy sets. Fuzzy sets have been defined as sets whose elements have degree of membership. It was originally proposed by Lotfi A. Zadeh and Dieter Klaua in 1965.

Fuzzy sets were further extended to Intuitionistic Fuzzy Sets (IFS) which are the sets whose elements have degree of membership as well as non-membership. Intuitionistic Fuzzy Sets were introduced by Krassimir Atanassov in 1983. IFSs can be applied to many fields but some of its common applications are:

- 1) Image Stabilization and Image processing with the help of fuzzy sets.
- 2) It is used in aerospace industry like altitude control of spacecraft and satellites, flow and mixture regulation in aircraft deceiving vehicles.
- 3) It can be applied in the automotive industry also like to find the shift scheduling methods for automatic transmission, how to increase fuel efficiency in automatic transmission etc.
- 4) Fuzzy sets are used in Decision making support systems and personnel evaluation in large companies, banknote transfer control, fund management, stock market predictions.
- 5) In the medical industry it is implemented on a large scale in the following medical procedures like medical diagnostic support system, control of arterial pressure during anesthesia, multivariable control of anesthesia, modeling of neuropathological findings in Alzheimer's patients, radiology diagnoses, fuzzy inference diagnosis of diabetes and prostate cancer etc.
- 6) It is also used in industrial sector where it is implemented in the following techniques, Cement kiln controls (dating back to 1982), heat exchanger control, activated sludge wastewater treatment process control, water purification plant control, quantitative pattern analysis for industrial quality assurance, control of constraint satisfaction problems in structural design, control of water purification plants.

Apart from these applications, fuzzy sets have applications in other fields too like chemical industry, robotics, marine, mining and metal processing, securities, transportation etc.

PRE-REQUISITES

Decision Making

Decision making is very common activity that occurs frequently in every human functioning. It usually consists of selecting / finding the best option from a finite number of options in a giving situation. Decision making has received a great deal of interest from researchers and practitioners in many disciplines including engineering sociology, medical science, applied mathematics, economics, computer science and artificial intelligence etc. This is because effective and efficient decision making can substantially determine the profitability and even survival of individual organizations and directly improve the quality of human lives

Fuzzy Set

A fuzzy set \tilde{A} defined on a finite universe of discourse $X = \{x_1, x_2, \dots, x_n\}$ is given by

$$\tilde{A} = \{ \langle x, \mu_{\tilde{A}}(x) \rangle \mid x \in X \},$$

where $\mu_{\tilde{A}} : X \rightarrow [0,1]$ is the membership function of \tilde{A} .

The membership value $\mu_{\tilde{A}}(x)$ describes the degree of belongingness of $x \in X$ to the set \tilde{A} .

Attanassov, as mentioned earlier, generalized the idea of fuzzy sets, to what is called intuitionistic fuzzy sets, defined as follows:

Intuitionistic Fuzzy Set

An intuitionistic fuzzy set A defined on a finite universe of discourse $X = \{x_1, x_2, \dots, x_n\}$ may be mathematically defined as

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \},$$

where

$$\mu_A : X \rightarrow [0,1] \text{ and } \nu_A : X \rightarrow [0,1]$$

with the condition

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1, \quad \forall x \in X.$$

The numbers $\mu_A(x)$ and $\nu_A(x)$ denote respectively the *degree of membership* and *degree of non-membership* of $x \in X$ to the set A .

For each IFS A in X , if $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x), x \in X$, then $\pi_A(x)$ represents the degree of hesitance of $x \in X$ to the set A , $\pi_A(x)$ is also called *intuitionistic index*.

Obviously, when $\pi_A(x) = 0$, i.e., $\nu_A(x) = 1 - \mu_A(x)$ for every x in X , then the IFS set A reduces to a fuzzy set. Thus, fuzzy sets are the special cases of IFSs.

Let $IFS(X)$ denote the family of all IFSs in the universe X , and let $A, B \in IFS(X)$ be given by

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \},$$

$$B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle \mid x \in X \}.$$

then usual set relations and operations are defined as follows:

$$A \subseteq B \text{ iff } \mu_A(x) \leq \mu_B(x) \text{ and } \nu_A(x) \geq \nu_B(x) \quad \forall x \in X$$

$$A = B \text{ iff } A \subseteq B \text{ and } B \subseteq A;$$

$$A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle \mid x \in X \}$$

$$A \sqcap B = \{ \langle \mu_A(x) \wedge \mu_B(x) \text{ and } \nu_A(x) \vee \nu_B(x) \rangle \mid x \in X \}$$

;

$$A \sqcup B = \{ \langle \mu_A(x) \vee \mu_B(x) \text{ and } \nu_A(x) \wedge \nu_B(x) \rangle \mid x \in X \}$$

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Let $\Delta_n = \{ P = (p_1, p_2, \dots, p_n) : p_i \geq 0, \sum_{i=1}^n p_i = 1 \}, n \geq 2$ be a set of n -complete probability distributions.

Kerridge's inaccuracy of distribution $Q = (q_1, q_2, \dots, q_n) \in \Delta_n$ with respect to distribution $P = (p_1, p_2, \dots, p_n) \in \Delta_n$ is given by

$$I(P; Q) = - \sum_{i=1}^n p_i \log q_i \tag{1}$$

This obviously can be seen as the average of information elements, namely, $-\log q_i$'s, of distribution Q over a distribution P , in a sense generalizing Shannon's entropy.

Intuitionistic Fuzzy Inaccuracy

Given two intuitionistic fuzzy sets A and B defined on a discrete universe of discourse $X = \{x_1, x_2, \dots, x_n\}$ with membership values $\mu_A^i, \mu_B^i, i=1, 2, \dots, n$, and non-membership values $\nu_A^i, \nu_B^i, i=1, 2, \dots, n$, the intuitionistic fuzzy inaccuracy of an intuitionistic fuzzy set B with respect to intuitionistic fuzzy set A , is defined as

$$I_{IFS}(A; B) = -\frac{1}{n} \sum_{i=1}^n \left[\begin{aligned} &\mu_A^i \log \frac{\mu_A^i + \mu_B^i}{2} + \nu_A^i \log \frac{\nu_A^i + \nu_B^i}{2} \\ &+ \pi_A^i \log \frac{\pi_A^i + \pi_B^i}{2} - \pi_A^i \log \pi_A^i \\ &- (1 - \pi_A^i) \log (1 - \pi_A^i) - \pi_A^i \end{aligned} \right]$$

APPLICATION

Let us consider the data [7, 22, 23, 33, -36]. Suppose that there are four patients Al, Bob, Joe, Ted,

represented as $P = \{Al, Bob, Joe, Ted\}$. Their symptoms are $S = \{\text{Temperature, Headache, Stomach pain, Cough, Chest pain}\}$. The set of diagnoses is defined as $D = \{\text{Viral fever, Malaria, Typhoid, Stomach problem, Chest problem}\}$. The intuitionistic fuzzy relation $P \rightarrow S$ is presented in Table 3. Table 4 gives the intuitionistic fuzzy relation $S \rightarrow D$. Each element of the tables is given in the form of IFV, which is a pair of numbers corresponding to the membership and

Non-membership values, respectively. In order to make a proper diagnosis for each patient, we calculate the similarity degree between each patient and each diagnose. According to the principle of maximum similarity degree, the higher similarity degree indicates a proper diagnosis. In Table 5, the similarity degree SY between patients and diagnoses is presented. According to the similarity degrees in Table 5, conclusion can be made that Al suffers from Viral fever, Bob suffers from Stomach problem, Joe suffers from Typhoid, and Ted suffers from Viral fever. The diagnosis results for this case obtained in previous study have been presented in . It is clear that our proposed method provides the same results obtained by Vlachos and Sergiadis. Moreover, our proposed similarity measure is calculated based on the IFNs, without any other parameters such as p, t . So it can reduce the computation complexity.

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CONCLUSION

Since most of the existing similarity measures for IFSs have provided counterintuitive results, a new similarity measure and weighted similarity measure between IFSs were proposed in this paper. The new similarity measure is calculated based

on the membership degree $\mu_A(x)$, non-membership degree $\nu_A(x)$, hesitancy degree $\pi_A(x)$ and the upper bound of membership $1 - \nu_A(x)$. In some special cases where some of the existing similarity measures cannot provide reasonable results, the proposed similarity measure shows great capacity for discriminating IFSs. Moreover, investigation of the new measure's classification capability was carried out based on two numerical examples and medical diagnosis. It has been illustrated that the proposed similarity measure performs as well as or better than previous measures. Further research will be focused on its applications in other practical fields.

TABLE 3: Symptoms characteristic for the patients.

	Temperature	Headache	Stomach pain	Cough	Chest pain
Al	(0.8, 0.1)	(0.6, 0.1)	(0.2, 0.8)	(0.6, 0.1)	(0.1, 0.6)
Bob	(0, 0.8)	(0.4, 0.4)	(0.6, 0.1)	(0.1, 0.7)	(0.1, 0.8)
Joe	(0.8, 0.1)	(0.8, 0.1)	(0.0, 0.6)	(0.2, 0.7)	(0.0, 0.5)
Ted	(0.6, 0.1)	(0.5, 0.4)	(0.3, 0.4)	(0.7, 0.2)	(0.3, 0.4)

TABLE 4: Symptoms characteristic for the diagnoses.

	Viral fever	Malaria	Typhoid	Stomach problem	Chest pain problem
Temperature	(0.4, 0.0)	(0.7, 0.0)	(0.3, 0.3)	(0.1, 0.7)	(0.1, 0.8)
Headache	(0.3, 0.5)	(0.2, 0.6)	(0.6, 0.1)	(0.2, 0.4)	(0, 0.8)
Stomach pain	(0.1, 0.7)	(0.0, 0.9)	(0.2, 0.7)	(0.8, 0.0)	(0.2, 0.8)
Cough	(0.4, 0.3)	(0.7, 0.0)	(0.2, 0.6)	(0.2, 0.7)	(0.2, 0.8)
Chest pain	(0.1, 0.7)	(0.1, 0.8)	(0.1, 0.9)	(0.2, 0.7)	(0.8, 0.1)

TABLE 5: The proposed similarity measure S_y between each patient's symptoms and the considered set of possible diagnoses.

	Viral fever	Malaria	Typhoid	Stomach problem	Chest problem
Al	0.9347	0.9228	0.9223	0.7673	0.7490
Bob	0.8124	0.6775	0.8997	0.9760	0.8211
Joe	0.9152	0.8271	0.9188	0.7917	0.7456
Ted	0.9576	0.9034	0.9060	0.8577	0.8122

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